

# Core 3 - June 2007

① a)  $y = \ln(x)$        $dy/dx = 1/x$

b)  $y = (x+1) \ln(x)$        $u = x+1$        $v = \ln(x)$   
 $dy/dx = \ln(x) + 1/x(x+1)$        $du/dx = 1$        $dv/dx = 1/x$   
 $= \ln(x) + 1 + 1/x$

Product Rule

c)  $x = 1$

$y = 2 \ln(x) = 0$

$dy/dx = \ln(x) + 1 + 1/x$

At  $x=1$        $= \ln(1) + 1 + 1 = 2$        $\Rightarrow$  gradient of normal  $= -1/2$   
gradient

$y - y_1 = m(x - x_1)$

$y - 0 = -1/2(x - 1) \Rightarrow y = -1/2(x - 1)$

② a)  $y = (x-1)^4$        $dy/dx = 4(x-1)^3$

b)  $y^2 = [2\sqrt{(x-1)^3}]^2 = 4(x-1)^3$

Volume  $= \pi \int_2^4 y^2 dx$

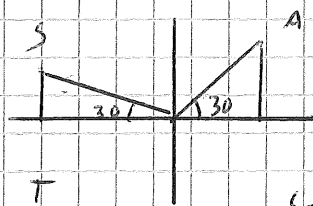
$= \pi \int_2^4 4(x-1)^3 dx = \pi [(x-1)^4]_2^4$

$= \pi [3^4 - 1^4] = 80\pi$

c)  $(x-1) =$  Translation  $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

2 = stretch, SF 2, parallel to y axis

③ a)  $\cos(x) = 2 \Rightarrow \sin(x) = 1/2$

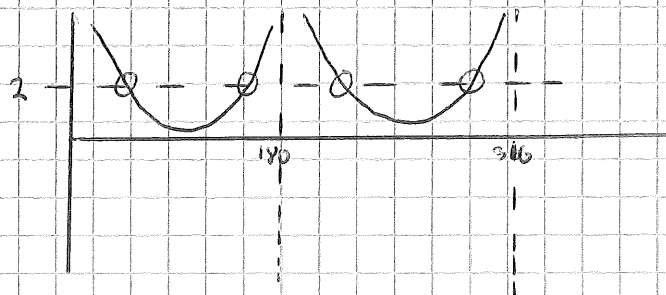


$x = 30^\circ, 150^\circ$

b) i)  $[90, 1]$

c) By symmetry:

ii)



$x = 30, 150$

$210, 330$

4) a)

$x_0$	1	3
$x_1$	1.25	3.9482...
$x_2$	1.5	5.1962...
$x_3$	1.75	6.8385...
$x_4$	2	9

$h = 0.25$

$$A = \frac{1}{3} \times 0.25 \times (3 + 9 + 4(3.9482 + 6.8385) + 2(5.1962)) = 5.46 \text{ (3sf)}$$

b) i) At intersection:  $3^x = x + 3 \rightarrow 3^x - x - 3 = 0$

Let  $f(x) = 3^x - x - 3$

$f(0.5) = -1.7679...$

$f(1.5) = 0.69615...$

Sign change,  $\therefore$  root lies between 0.5 and 1.5

ii)  $3^x = x + 3$

$x \ln(3) = \ln(x+3) \rightarrow x = \frac{\ln(x+3)}{\ln(3)}$

iii)  $x_1 = 0.5$

$x_2 = \frac{\ln(0.5+3)}{\ln(3)} = 1.1403...$

$x_3 = \frac{\ln(1.1403+3)}{\ln(3)} = 1.2932... = 1.3 \text{ (2sf)}$

iv) See Insert

5) a)

$f(x) = \sqrt{x-2}$

Range:  $f(x) \geq 0$

cannot be negative!

b) i)  $f(g(x)) = f(\frac{1}{3}x) = \sqrt{\frac{1}{3}x - 2}$

ii)  $\sqrt{\frac{1}{3}x - 2} = 1 \rightarrow \frac{1}{3}x - 2 = 1$

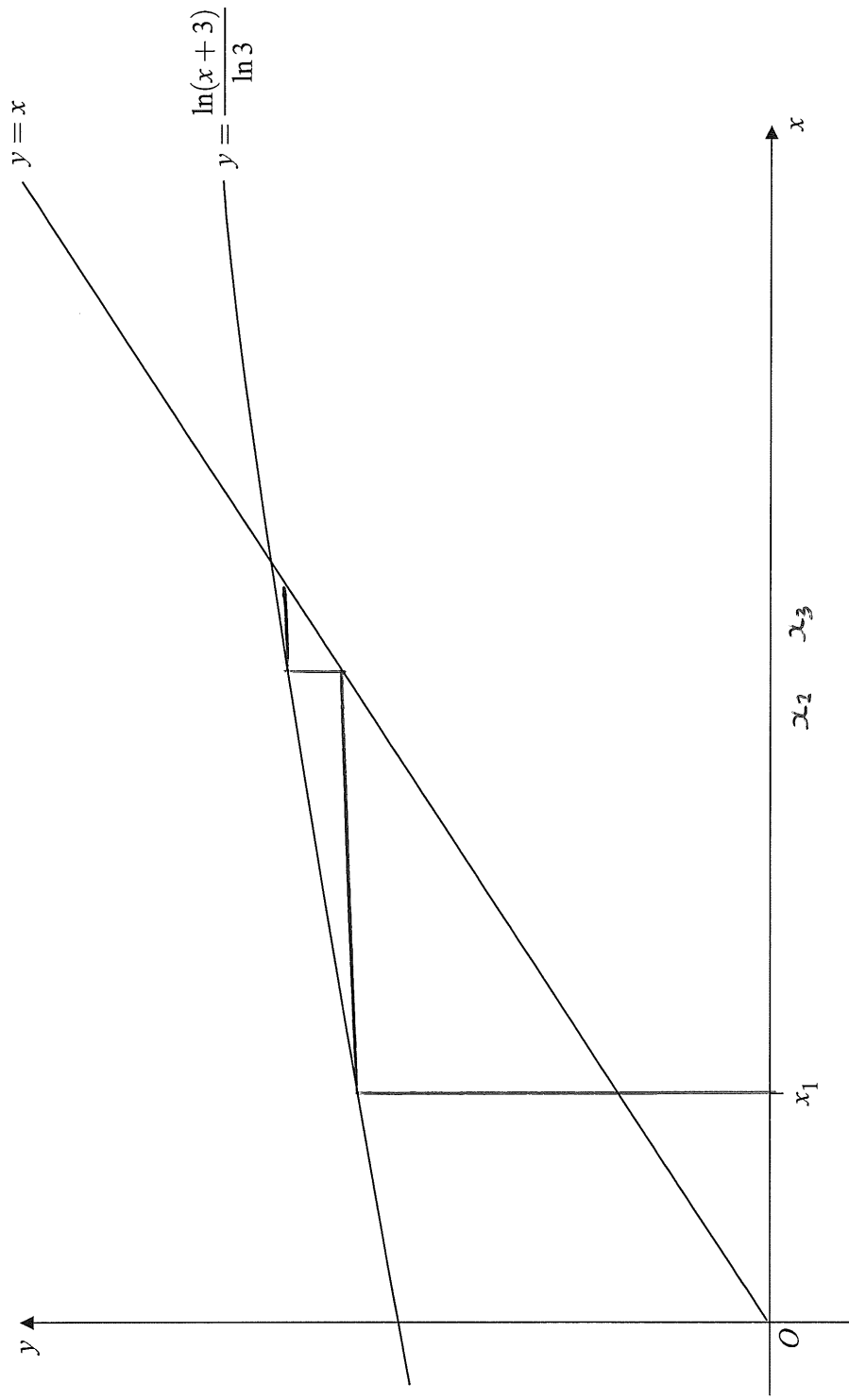
$\frac{1}{3}x = 3 \rightarrow x = 9$

c) Let  $y = \sqrt{x-2}$

$y^2 = x - 2$

$x = y^2 + 2 \rightarrow y = \sqrt{x-2} = f^{-1}(x)$

Figure 1 (for use in Question 4)



6) a)  $\int x e^{5x}$

$u = 5x$

$\frac{dv}{dx} = e^{5x}$

$\frac{du}{dx} = 5$

$v = \frac{1}{5} e^{5x}$

$= uv - \int \frac{dv}{dx} u dx$

$= \frac{x}{5} e^{5x} - \int \frac{1}{5} e^{5x} dx$

$= \frac{x}{5} e^{5x} - \frac{1}{25} e^{5x} + C$

b) i)  $\int \frac{1}{\sqrt{x}(1+\sqrt{x})} dx$

$u = \sqrt{x} = x^{1/2}$

$\frac{du}{dx} = \frac{1}{2} x^{-1/2} = \frac{1}{2\sqrt{x}}$

$= \int \frac{2\sqrt{x}}{\sqrt{x}(1+u)} du$

$\rightarrow dx = 2\sqrt{x} du$

$= \int \frac{2}{1+u} du$

ii) change limits:  $u = \sqrt{x}$      $1 \rightarrow 1$ ,  $4 \rightarrow 3$

$\int_1^3 \frac{2}{1+u} = [2 \ln(1+u)]_1^3$

$= 2 \ln(4) - 2 \ln(2)$

$= 2 \ln(2) = \ln(4)$

7) a)  $y = (x^2 - 3)e^x$

Product Rule

i)  $\frac{dy}{dx} = 2xe^x + e^x(x^2 - 3)$

$u = x^2 - 3$	$v = e^x$
$\frac{du}{dx} = 2x$	$\frac{dv}{dx} = e^x$

ii)  $\frac{d^2y}{dx^2} = 2e^x + 2xe^x + 2xe^x + e^x(x^2 - 3)$

$u = 2x$	$v = e^x$
$\frac{du}{dx} = 2$	$\frac{dv}{dx} = e^x$

b) i) At st point,  $\frac{dy}{dx} = 0$

$2xe^x + e^x(x^2 - 3) = 0$

$e^x(2x + x^2 - 3) = 0$

$e^x(x^2 + 2x - 3) = 0$

$e^x(x + 3)(x - 1) = 0$



ii)  $\boxed{x=1}$

$\frac{d^2y}{dx^2} = 2e^1 + 2e^1 + 2e^1 + e(-2)$   
 $= 10.873$   
 $= \text{MINIMUM}$

$\boxed{x=-3}$

$\frac{d^2y}{dx^2} = -0.2$   
 $= \text{MAXIMUM}$

8) a)  $\int \sec^2(x) dx = \tan(x) + c$

b)  $y = \frac{\cos(x)}{\sin(x)}$

$u = \cos(x)$

$v = \sin(x)$

$\frac{du}{dx} = -\sin(x)$

$\frac{dv}{dx} = \cos(x)$

$\frac{dy}{dx} = \frac{-\sin^2(x) - \cos^2(x)}{\sin^2(x)}$

$= \frac{-1(\sin^2(x) + \cos^2(x))}{\sin^2(x)} = \frac{-1}{\sin^2(x)} = -\cos^2(x)$

c)  $(\tan(x) + \cot(x))^2 = \tan^2(x) + 2\tan(x)\cot(x) + \cot^2(x)$   
 $= \tan^2(x) + 2\left(\frac{\sin(x)}{\cos(x)} \times \frac{\cos(x)}{\sin(x)}\right) + \cot^2(x)$   
 $= \tan^2(x) + 2 + \cot^2(x)$   
 $= \tan^2(x) + 1 + \cot^2(x) + 1$   
 $= \sec^2(x) + \operatorname{cosec}^2(x)$

d)  $\int_{0.5}^1 \sec^2(x) + \operatorname{cosec}^2(x)$   
 $= \left[ \underset{\substack{\uparrow \\ \text{From a)}}}{\tan(x)} + \underset{\substack{\uparrow \\ \text{From b)}}}{-\frac{\cos(x)}{\sin(x)}} \right]_{0.5}^1$   
 $= \tan(1) - \frac{\cos(1)}{\sin(1)} - \tan(0.5) + \frac{\cos(0.5)}{\sin(0.5)}$   
 $= 0.91531... - 1.28418...$   
 $= 2.1992...$   
 $= 2.2 \text{ (2sf)}$